Solving Radical Equations Algebraically & Graphically

Strategies for Solving Graphically

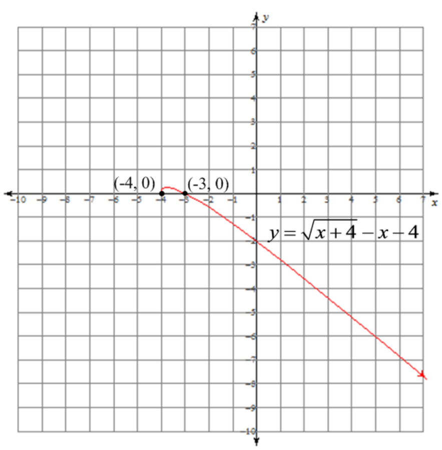
Method 1: Use a Single Function

Rearrange the radical equation so that one side is equal to zero. Graph the corresponding function and find the x-intercepts of the graph.

Example: Solve $2+\sqrt{x+4}=x+6$

Rearrange: $\sqrt{x+4}-x-4=0$

Graph: $y=\sqrt{x+4}-x-4$



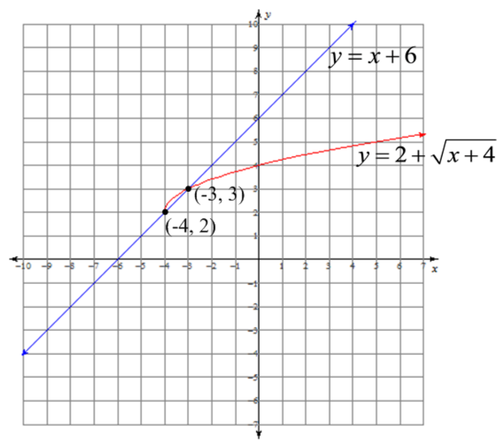
Solutions: x = –3 or x = –4

Method 2: Use a System of Two Functions

Express each side of the equation as a function. Graph these functions and determine the value of x at the point(s) of intersection.

Example: Solve $2+\sqrt{x+4}=x+6$

Graph: $y=2+\sqrt{x+4}$ and $y=x+6$



Solutions: x = –3 or x = –4

Strategy for Solving Algebraically

Step 1: Isolate the radical.
Step 2: Square both sides of the equation to eliminate the radical.
Step 3: Continue to solve for x.
Step 4: Check for extraneous solution(s).

Example: Solve $2+\sqrt{x+4}=x+6$

$$2+\sqrt{x+4}=x+6$$
$$\sqrt{x+4}=x+4$$
$$x+4=x^2+8x+16$$
$$0=x^2+7x+12$$
$$0=(x+3)(x+4)$$
$$x=-3 \text{ or } x=-4$$

Check $x=-3$:

$$LHS=2+\sqrt{-3+4}=2+\sqrt{1}=3$$
$$RHS=-3+6=3$$
$$LHS=RHS$$

Check $x=-4$:

$$LHS=2+\sqrt{-4+4}=2+0=2$$
$$RHS=-4+6=2$$
$$LHS=RHS$$

Example 1: Relate Roots and x-Intercepts

For the radical equation $2\sqrt{x-4}-3=0$:

- Algebraically determine the root(s). State any restrictions on the variable.
- Graph the corresponding function (method 1) and determine the x-intercepts.
- Describe the connection between the root(s) of the equation and the x-intercept(s) of the graph of the corresponding function.

Solution:

Algebraically	Graphically
$2\sqrt{x-4}-3=0$ Restrictions: Solve:: Solution(s): ____________________ Check:	$2\sqrt{x-4}-3=0$ Using technology, graph the corresponding function $y=2\sqrt{x-4}-3$ and determine the x-intercept(s). x-intercept(s): ________________________ Solution(s): _______________________

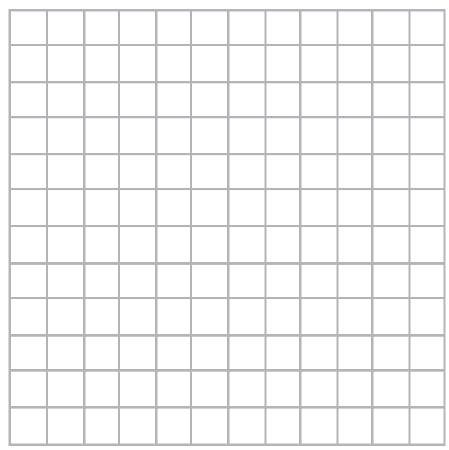
The root(s), or solution(s), of a radical equation are equal to the ____________________ of the graph of the corresponding function.

Example 2: Solve a Radical Equation Involving an Extraneous Root

For the equation $\sqrt{x+5} = x+3$:

- Algebraically determine the root(s). State any restrictions on the variable.
- Graph the corresponding functions (method 2) and determine the point(s) of intersection.
- Describe the connection between the root(s) of the equation to the point(s) of intersection of the two functions.

Solution:

Algebraically	Graphically
$\sqrt{x+5} = x+3$ Restrictions: Solve: Solution(s): ____________________ Check:	$\sqrt{x+5} = x+3$ Using technology, graph the corresponding functions $y = \sqrt{x+5}$ and $y = x+3$ and determine the point(s) of intersection of the two graphs.  Point(s) of intersection:____________________ Solution(s): ____________________

The two functions intersect at the point __________. The value of x at this point, __________, is the solution to the equation.